CHAPTER 1

Over 50 Years of Image Moments and Moment Invariants

George A. Papakostas

This chapter aims to analyze the research field of moments and moment invariants in a holistic way. Initially, a literature analysis of the last 50 years is presented and discussed in order to highlight the potential of this topic and the increasing interest in many disciplines. A more in depth study of the issues addressed through the years by the researchers is next presented both in theory and applications of moments. The most representative works in each research direction are discussed in a chronological order to point out the progress in each specific field of action. This analysis concludes with the challenges and perspectives that should motivate researchers towards the promotion of the moments and their invariants to new scientific "horizons".

For the first time, this chapter gives a global overview of what happened in the last 50 years in moments and moment invariants research field, but most of all it brings to light the open issues that should be addressed and highlights the rising topics that will occupy the scientists in the coming years. This chapter serves as a guide to those who find the field of moments and moment invariants a "brilliant field of action" since it encloses all the milestones of this field.

George A. Papakostas
Department of Computer and Informatics Engineering
Eastern Macedonia and Thrace Institute of Technology (EMaTTech)
65404 Kavala, Greece
e-mail: gpapak@teikav.edu.gr

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1.1 Introduction

The first introduction of 2-D moments in engineering life was performed by Hu in 1962 [60]. Hu proposed the 2-D geometric moments of a distribution function (an image) as a structured element of what he called "moment invariants". In that work, Hu used the theory of algebraic invariants in order to define seven orthogonal invariants to linear transformations (translation, rotation, scaling, skew).

Since then, after more than 50 years of research, a lot of new achievements in the theory of moments and moment invariants have been presented. The resulted new theoretical framework has boosted the applicability of moments in many disciplines, while a continuously increasing number of scientists have set the moments in the center of their research.

The next milestone was the introduction of orthogonal moments by Teague [150] in 1980. Teague proposed Zernike and Legendre orthogonal moments in image analysis as a solution to the inherent drawback of geometric moments and Hu's invariants too, the high information redundancy. Geometric moments are the projection of the intensity function of an image onto specific monomials, which do not construct an orthogonal basis. Orthogonal moments came to overcome this disadvantage of the conventional moments since their kernels are orthogonal polynomials. The property of orthogonality gives to the corresponding moments the feature of minimum information redundancy, meaning that different moment orders describe different part of the image.

The first detailed analysis of moments' properties and performance in image analysis was performed by Teh and Chin's [151]. This analysis had inspired all the later works in orthogonal moments and helped to understand the power of describing an image in terms of an orthogonal polynomials' base. As a consequence of Teh and Chin [151] work was the introduction of Zernike moments in pattern recognition by Khotanzad and Hong [71]. Since then Zernike moments constituted the most popular moment family due to their inherent property of being invariant under rotation and flipping of the image.

Belkasim et al. [8] investigated the performance of the moment invariants by comparing the algebraic and orthogonal moment invariants proposed until then, in pattern recognition applications. This comparative study resulted to a new set of Zernike moment invariants derived by proper normalization of the corresponding moments in order to reduce their range.

The next milestone was the introduction of affine moment invariants by Flusser and Suk [33]. They extended Hu's algebraic moment invariants to general affine transformations by proposing four affine moment invariants. The significance of affine moment invariants was their capability to recognize affine-deformed patterns commonly occurring in real pattern recognition problems such as character recognition and shape classification.

The following 20 years moments and moment invariants have attracted the attention of scientists towards the construction of new moment families [47, 49, 50, 43, 96, 121, 182, 186, 201, 199], the improvement of the computation accuracy [83, 95, 108, 52, 168], the development of fast computation algorithms [97, 21, 116, 53], the embodiment of invariant properties to the moment functions[7, 16, 22, 45, 69], etc. On the other hand an increased number of applications have been shown suitable for applying moments and moment invariants such as image analysis [203, 202, 20], pattern recognition [1, 44, 94, 103, 105, 111, 66], multimedia watermarking [3, 72, 177, 165, 158], image retrieval [170, 136, 137], medical image analysis [24, 87, 173], forensics [89, 38, 86, 131], etc.

This chapter aims to provide an overview of the research in the field of moments and moment invariants since the first introduction of moment invariants by Hu [60]. Initially, a literature analysis of the last 50 years is presented and discussed in order to highlight the high potential of this topic. Moreover, this chapter provides an in depth study of the issues that have been addressed through the years by the researchers both in theory and applications of moments. The most representative works in each research direction are discussed in a chronological order to point out the progress in each specific field of action. This analysis concludes with the challenges and perspectives that should motivate the researchers towards the promotion of the moments and their invariants to new scientific "horizons". This chapter aims to serve as a guide to those who are interested in knowing the evolution of moments and moment invariants and to promote the current scientific achievements to the next levels.

The chapter is organized as follows: Section 1.2 briefly discusses the background of moments' definitions by providing the necessary information regarding the moment types and their corresponding properties. Section 1.3 presents a detailed literature analysis by discussing the publication activity in the field of moments and moment invariants during the last 50 years. Section 1.4 summarizes all the attempts towards the dissemination of knowledge about the moments such as books and organized events. Section 1.5 reviews the directions towards theory and applications that scientists have focused their research on, by discussing the reasons which generated each need, the current state and the future challenges of each issue. Section 1.6 determines the big challenges for the community of moments and finally Section 1.7 concludes the overall chapter by highlighting the most important discussed issues.

1.2 Background

Before proceeding with the demonstration and discussion of the research directions in moments and moment invariants, it is constructive to give a short introduction to the fundamentals of moment functions.

Traditionally, the orthogonal image moments are considered as statistical quantities that describe the pixels distribution inside an image's space. Mathematically, they are computed as the projections of an image to the orthogonal basis of the used polynomials. From an engineering and computer science point of view, the orthogonal moments represent the similarity between the image and a number of image patterns formed by the kernel function of each moment family [101].

1.2.1 Moment Functions Taxonomy

The most straightforward way to classify the moment functions is based on their dimension (number of variables). Thus, there are 1-D , 2-D and 3-D moment functions applied on signals (one dimensional), images and volumes respectively, as illustrated

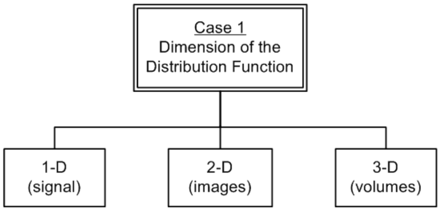

Figure 1.1: Moment functions taxonomy: Case 1 - Dimension of the distribution function.

in Fig.(1.2.1).

Although the moment functions can be of any number of variables, herein we are interested in the 2-D moments of an image. Moreover, even though there are orthogonal and non-orthogonal moment types the following presentation is restricted to orthogonal moments due to their popularity.

The moment functions are characterized by the type of the polynomials base, resulting to a number of different moment types with specific properties. However, there are several other more general characteristics that can define the moments taxonomy. Moments are classified to continuous and discrete, whereas their coordinate space is the continuous real space or the the discrete space of the image. This case of moments classification, regarding the type of the coordinate space along with some representative moment families of each type, is depicted in Fig.(1.2). Since we are interested in the moments of an image intensity function the continuous moments should be transformed (zero-th order approximation) to a form suitable for applying on the image.

The general computation form of the $(p+q)$-th order of any moment type and of an image intensity function $f(x,y)$ of $N \times N$ pixels size is defined as:

$$M_{pq} = NF \times \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} Kernel_{pq}(x,y) f(x,y),$$

where $Kernel_{pq}(\cdot)$ corresponds to the moment's kernel consisting of the product of the specific polynomials [116, 101] of order p and m , which constitute the orthogonal basis and NF is a normalization factor. The type of Kernel's polynomial gives the name to the moment family by resulting to a wide range of moment types Fig.(1.2).

Recently, there is an increased interest in the computation of color images [17, 69, 68, 15], by making the already proposed methods for the gray-scale images inappropriate for being applied. Therefore, the moments can be categorized according to the depth of the intensity function 1.3 to 8-bit (gray-scale) and 24-bit (color) moment functions.

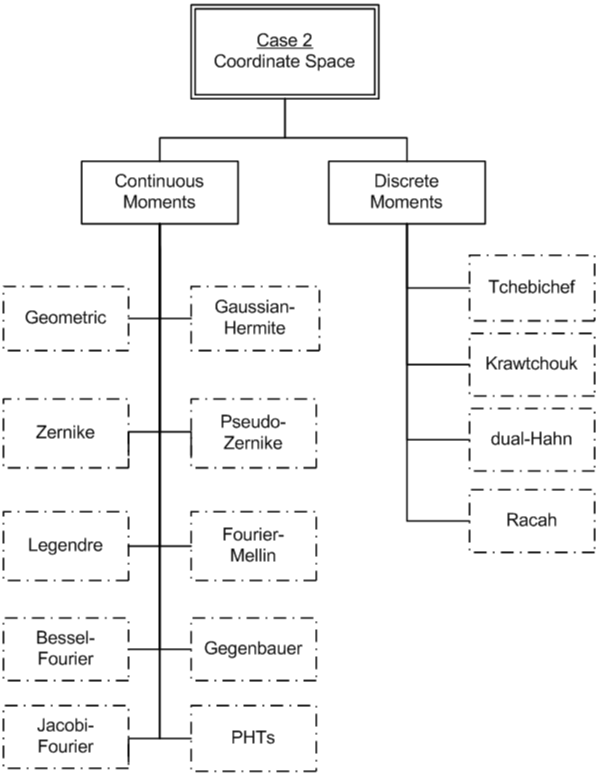

Figure 1.2: Moment functions taxonomy: Case 2 - Coordinate space.

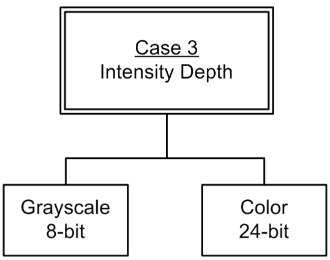

Figure 1.3: Moment functions taxonomy: Case 3 - Intensity depth.

1.2.2 Moment Invariants

Although moments are efficient descriptors of the image's content, they are sensitive to several geometric (rotation, translation, scaling, affine) and non-geometric (blur) transformations of the image. In order to alleviate this shortcoming scientists have proposed their invariants called *moment invariants*. These values have the same properties as the corresponding moments and they are robust to several image deformations. The moment invariants [8] are widely used as discrimination features in pattern recognition and classification applications.

1.3 Literature Analysis

As it is stated in the introduction section, the main goals of this chapter is the justification of the continuously increased scientific interest in moments and moment invariants, the presentation of the research directions showing considerable activity and the declaration of the research actions that should be followed in order to further improve and spread the multi-discipline utilization of moments and their invariants. While the last two objectives will be satisfied via a briefly description of what has been done and what needs to be done in the field of moments, the former objective can be achieved by analyzing the publications related to moments in the literature through the years.

The analysis of the literature in order to find and count the number and type of publications during a certain period constitutes a laborious task. However, for this study it is decided to make use of the well known *Scopus* bibliographic database [25], which is commonly accepted by the scientific community and includes enough information for our analysis. The searching has been performed by applying the keywords *moments*, *image*, and *moment invariants*, which are necessary to appear in the Title, Abstract, and Keywords sections of the publications.

The period of our analysis was set from the introduction of moment invariants by Hu [60] in 1962 to the current year 2014, although the publication activity of latter year is still in progress. Moreover, we are interested only in three types of publications namely, Journals, Book Chapters and Conference papers. In the hereafter results the first two types are tackled as a single one since the number of book chapters is quite small.

Figure 1.4, illustrates the number of papers published in the last 50 years, where the period 1962-1980 is merged due to the very small number of published studies, in time step of 5 years. From this plot, the upward trend of the interest in moments and moment invariants is obvious with the 5-year period 2005-2010 being characterized by the rapid increase of all types of publications. Moreover, the importance of moments' field is justified by the publication of more journals and book chapters, known for the more rigorous review process, than conferences.

By focusing our analysis on the time period of the last 10 years it can be also derived that 2009 was the most productive year in the history of moments and moment invariants, during which 1,231 papers of all types have been published. This number is very big considering the high competitiveness taking place in the field of image description and reflects the amplification of the engagement of new scientists with the

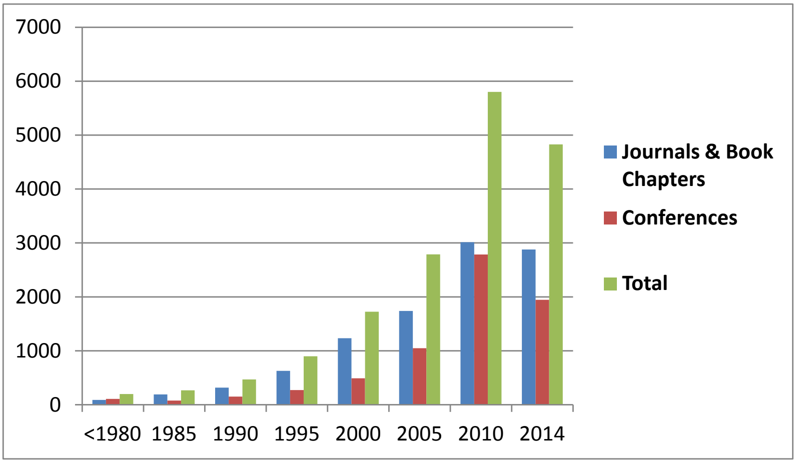

Figure 1.4: Moments related publications for the past 50 years (per 5 years).

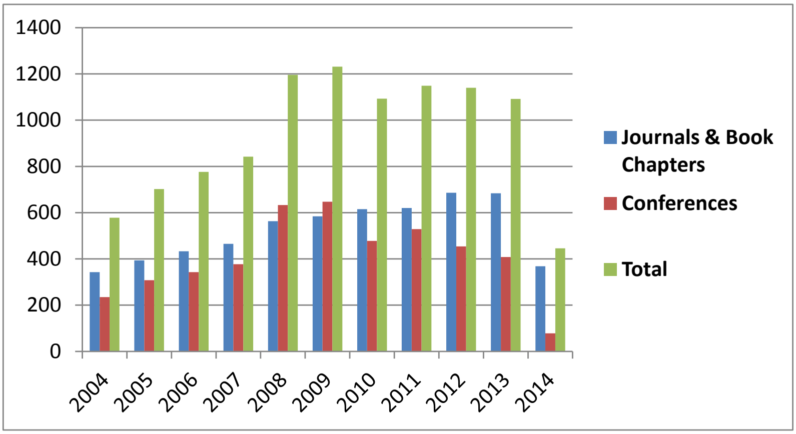

Figure 1.5: Per year moments related publications for the last 10 years.

moments related topics. In the following years the publishing activity has attained the same levels with 2009, with the number of journals and book chapters showing an upward trend.

Conclusively, can be stated that the research in the field of moments experiences its highest evolution so far. The outcomes of this study should be translated to more research activities, since time and high prior-knowledge favor the discovering and developing of the next generation frameworks in both moments' theory and applications.

1.4 Knowledge Dissemination

Apart from the publications in international journals and conferences the dissemination of knowledge regarding moments and moment invariants, some other types of actions have been performed in the past too. Towards this direction three popular books have been published until now, which permit the early stage researchers in this topic to study the basics of moments and moment invariants.

The first book was written by Mukundan and Ramakrishnan [98] in 1998 and constituted the only text for the researchers for about 10 years. This book summarizes the main theoretical aspects of several moment functions, with emphasis to their application in image analysis. Moreover, it discusses and proves analytically some useful properties of the moment functions and reviews the main publications proposed until then.

The second book was written by Pawlak [120] in 2006. This book provides a different approach to the moment theory from the previous book, since it focuses on the reconstruction performance of the moment functions, with emphasis to their computation accuracy. The advantage of this book is that it is available [120] for free downloading and thus anyone can retrieve it.

The third book was written by Flusser et al. [36] in 2009, only 3 and 10 years after the Pawlak's and Mukundan's books respectively. The mentioned time span is significant since it gives us an indication of the differences between the books' contents and thus the amount of novel information they contain. The third book can be compared only with the Mukundan's book, since it constitutes its update version enriched with the theory developed in the meanwhile and with emphasis to pattern recognition applications. These differences make the third book the current textbook for any researcher in this research field.

Finally, for the dissemination of moments a special session in the International Conference of Image Analysis and Recognition (ICIAR) was organized recently by Al-Rawi et al. [2]. However, the small number of contributed papers and its temporary nature have shown that additional efforts should be made in collaboration with the most eminent scientists in this field in order to establish a frequent annual event (e.g. special session or workshop) as the major meeting for the moments' knowledge dissemination.

1.5 Research Directions

Although the moment functions and their invariants were introduced almost 50 years ago, their evolution was extremely significant only in the last 20 years. The development of novel theoretical frameworks have boosted the disciplines of their application. In this section, it is attempted to present some important snapshots of progress in both moments' theory and applications.

1.5.1 Theory

It is well known that the dissemination of a specific research field is highly dependent on the amount and accuracy of the theoretical framework that supports its scientific correctness. Following the aforementioned trend and towards the alleviation of specific weaknesses and limitations of the fundamental moment theory, scientists have built new tools and methods. The most representative achievements in the theory of moments and moment invariants are summarized and highlighted in the hereafter subsections.

1.5.1.1 Fast Computation

Due to the fact that the computation of a moment or a moment invariant consists of the evaluation of the moment's base in each point of the distribution function (intensity function in the case of an image), the whole procedure is time consuming. The computation time is further increased in the case of the moments of an image (2-D or 3-D), since the moment's base should be evaluated across each dimension of the intensity function. Moreover, when a set of orthogonal moments is to be computed the computation time increases exponentially because of the high complexity of the polynomial basis. Therefore, the development of fast computation algorithms [116] were a primary target of the scientists for many years, while their achievements helped towards the computation of moments for big image data.
Mainly, the developed algorithms which ensure high computation speeds of image moments are divided into two different approaches: (1) *Polynomial level* and 2) *Pixels level*. Moreover, the latter approach is realized under three possible alternatives:

Approach 1 (Polynomial level) - The most common practice to reduce the time needed to compute the polynomial basis is to apply a recurrence formula to compute each polynomial order by using polynomials of lower orders. Several algorithms presented in the past introduced recursive algorithms which avoid the direct computation of the polynomials of any order, instead simplified recursive formulas were proposed. Such algorithms have been applied for the computation of Zernike [74, 122, 97, 21, 104, 106], Legendre [97], Fourier-Mellin [102, 162] moments etc. It is worth noting that some polynomials are equipped by recurrence computation formulas on their own such as Tchebichef [96], Krawtchouk [186], dual-Hahn [187, 201] Gaussian-Hermite [182] moments etc.

Approach 2 (Pixels level) - Another way to reduce the computation time of moments is to decrease the number of pixels where the polynomials are evaluated without loosing useful information. This can be achieved by either exploiting polynomials' symmetry properties (symmetric pixels have identical contribution to the moment calculation) or by treating the image as a set of intensity slices consisting of homogenous (with the same intensity value) rectangular blocks. According to the former strategy [96, 185, 61, 199] it is not necessary to compute the polynomial values for all symmetric pixels but only for a small portion of them, depending on the symmetry type. The latter strategy [145, 112, 115, 113, 134, 47] decomposes the moments computation to partial computations over rectangular homogenous blocks thus their moments can be derived easier. These two computation schemes show some limitations, the first strategy is applied only for those polynomials which exhibit symmetries and the second one can be applied only for polynomials defined in the Cartesian coordinate space. In this category can be also included an alternative computation scheme [53, 56]which is making use of the separability property of the moment transform permitting the computation of the moments in two steps, by successive computation of the corresponding 1-D moments for each row.

It is worth noting that some of the above algorithms can operate in combination [116, 113] under specific configurations in order to take advantage of each algorithm's acceleration capabilities. Furthermore, although these algorithms have been applied to compute the moments of an entire image, they can be used to compute the moments of image's partitions in a block-based computation scheme [116]. In this case, when image reconstruction is needed some partitioning effects occur, where each sub-image generates dark edges that degrade the overall reconstructed image quality.

Finally, following the recent trends of applying high performance hardware and software schemes for time consuming tasks, some researchers have proposed GPU based moments computation algorithms [153, 180, 64, 127]. Although, the introduced GPU accelerated algorithms show an improved computation speed, there is still room for more efficient accelerated algorithms that take advantage of the GPU resources (cores, threads, blocks, shared memory).

1.5.1.2 Computation Accuracy

According to the moments taxonomy presented in Section 1.2 the moments functions are classified to continuous and discrete in respect to the nature of their coordinate space. Only for the case of continuous moment functions, their computation over a the discrete pixels space of an image, encounters some inaccuracies. These errors are of two types namely *geometric* and *numerical* errors [83, 168, 178]. The first type of errors is caused by the projection of a square discrete image onto the domain (e.g. the unit disc for the radial polynomials) of the polynomial basis, while the numerical errors are occur due to the calculation of the double integral over fixed sampling intervals, by applying the zeroth order approximation.transformation.

Several approaches have been proposed in the literature towards the minimization of both error types. More precisely, the geometric errors are minimized by applying specific mapping techniques from the image space to the polynomials [168] domain and appropriate pixels arrangement methodologies [83]. The numerical integration errors are decreased by applying either analytical [168, 52, 56] or approximate iterative integration algorithms (e.g. Simpson, Gauss) [83, 55]. Currently, by using the aforementioned techniques the derived moment values are very close to their theoretical values and thus the achieved accuracy level is satisfactory.

1.5.1.3 Numerical Stability

Apart from the aforementioned computation errors, caused by the inherent weaknesses to apply the mathematical formulas to the set of image's pixels, the computation of image moments reveals some additional numerical instabilities [95, 108, 102, 109, 142, 143, 144, 126].

Recently, the author and his colleagues have analyzed the numerical behavior of the recursive algorithms for Zernike moments computation [108, 109]. They found that under certain circumstances some truncation errors called *finite precision errors* occur in an iteration of the algorithms, which is increased iteration by iteration and finally they cause the collapse of the algorithm. These errors are generated by specific mathematical operations such as subtraction and division.

The other common numerical instability occurring during the computation of image moments are the *overflows* [102, 106] due to the existence of big numbers and the great amount of operations between them. The overflows are more frequent with the increase of moment orders and the size of the image. This is the reason why the reconstruction of an image is considered only for small images ($< 1024 \times 1024$ pixels). Moreover, the presence of big quantities during the computation causes large variations in the dynamic range of moment values, [95]. In order to overcome this situation scaled moments were proposed[96, 185].

The numerical instabilities are responsible for the limitation of the maximum moment computed order along with the limitation to the size of the image being processed.

1.5.1.4 Invariance Embodiment

The introduction of moment invariants under the basic geometric transformation (translation, scale and rotation) by Hu [60], has shown the way to the most important application of moment function, to pattern/object recognition and data classification problems. The five Hu's invariants were based on the conventional geometric moments and their normalized versions. However, due to the description limitations (e.g. high information redundancy) of the geometric moments and the rising of the orthogonal moments, the moment invariants have been revised.

Mainly, there are two types of methodologies that derive invariant moments of an image, either by image coordinates normalization and description through the geometric moment invariants [98] or by developing new computation formulas which are characterized by invariant properties inherently [7, 22, 200]. In the first type of methods, we can also categorize the algebraic moment invariants since they use the geometric moments as a structural element.

Realizing the importance of describing and recognizing a scene/object/pattern despite the position, orientation, scale etc. of the region of interest inside an image, several moment invariants have been proposed: Affine [33, 148], Rotation [31, 32, 174], Geometric [179, 45], Orthogonal [185, 184, 69, 183], Blur [35, 193], Projective [189] moment invariants.

Recently, combined moment invariants which ensure invariant moment description under more than one geometric and non-geometric transformations, have been introduced [146, 197, 193, 16]. The combined invariants are very useful but they derive difficultly. The development of invariant moments under multiple geometric and non-geometric image transformations in combination, constitutes one of the hot topics in the field of moments.

Moreover, it is worth noting that a particular type of moment invariants called *Complete* moment invariants [39, 194] exhibiting some very useful properties, has been reported in the literature. The construction of a complete set of moment invariants is performed in terms of the corresponding moments of the same orders. This description has the advantage of enabling the inverse computation of the contributed moments from the corresponding invariants and vice versa (duality) .

1.5.1.5 Novel Moment Families

In the last 10 years there was an increased interest in developing new moment families and their corresponding invariants. As a result of this intense action is the introduction of new types of moments having improved properties as far as their description capabilities and invariance behavior are concerned.

In this context, the group of Geometric [60], Zernike [150], Pseudo-Zernike [150], Fourier-Mellin [133], Legendre [150] traditional moments, initially was enriched with the Tchebichef [96], Krawtchouk [185], dual-Hahn [201, 187], Racah [199], discrete moments exhibiting high computation accuracy. More moments such as Polar Harmonic Transforms [184, 49, 48] Wavelet [11, 117, 141], Gaussian-Hermite [182], Bessel-Fourier [175], Jacobi-Fourier [121, 12], Gegenbauer [82, 55], Charlier [46], Co-moments [189], Exponent [59], Variant [43] and Spline [19] moments were introduced in a way to find more informative and robust descriptors.

Recently, a new type of moments called *separable moments* [196, 47] was proposed. The separable moments are constructed by using a combination of different polynomials for each dimension. In this way the separable Chebyshev-Gegenbauer, Gegenbauer-Legendre, Tchebichef-Krawtchouk, Krawtchouk-Hahn etc. [196] and Charlier-{Tchebichef, Krawtchouk, Hahn} [47] were proposed. The separable have shown improved description capabilities compared to their non-separable versions, while their application in several disciplines constitutes a new field of action.

1.5.1.6 Color Moments

The increased usage of camera equipped devices in the every day life such as computers, mobiles, tablets etc. has triggered the need for efficient processing of color images. In the field of moments and moment invariants little work has been reported regarding the computation of moment function for color images.

The most straightforward practice is to compute the moments of each color channel separately [94, 147] and use them as 3-tuples for image analysis and recognition purposes. However, this approach has the disadvantages of applying the computation scheme for three times, by adding significant time overhead and the color information is not described in a compact way in a single number. Another similar approach is to first apply a color space transform (RGB to HSV) in order to isolate the color information into a single channel where the moments are being computed. In this case non-color useful information might be discarded.

However, in the last 2-3 years the perspective of applying quaternion analysis (a generalization of the complex analysis) in order to describe the color information of an image in a compact way has been promoted. The first introduced quaternion moments were the Quaternion Fourier-Mellin moments proposed by Guo and Zhu [42] in 2011. According to quaternion analysis, each image pixel is represented as a four-dimensional number called quaternions. After the first introduction of quaternion moments, the Quaternion Zernike [17, 15] and Quaternion Bessel-Fourier [132] moments and moment invariants [41, 93] were proposed. The author and his colleagues have developed a unified methodology [68, 70] to produce quaternion moments and moment invariants of any polynomial type, by giving scientists the option to derive the most appropriate to their applications color moments.

1.5.1.7 3D Moments and Moment Invariants

The evolution of the stereo imaging, by providing scene information in the three-dimensional (3-D) coordinate space, along with the development of cheap stereo cameras, has generated the need for moments computation of volumes. Although the concept of 3-D moments is not new [130, 13, 100], few works for handling 3-D moments and moment invariants have been reported. This is probably due to the small number of 3-D volumetric images that are incorporated in the every day life of humans. However, in the last years there is an increased interest in accelerating and improving the description capabilities of the 3-D moments, while novel 3-D moment families and moment invariants have been proposed [149, 54, 172].

What is worth investigating, is the way the previously introduced computation algorithms for 2-D images can be applied for the case of 3-D volumetric data. This research direction can lead to more unified computation schemes permitting the computation of the moments of any image modality.

1.5.1.8 Moments Selection

A common practice in using moments, is the computation of all moments up to a certain order and use them supposing that they describe adequately the image's content. However, this "ad hoc" usage of image moments is not optimal, in the sense that no prior knowledge regarding the problem at hand is taken into account, to guarantee the selected moments are the most appropriate for the specific task. A possible solution to this issue is the application of an additional process that selects, from a large pool, the moment features which best perform in terms of description accuracy (e.g. reconstruction error, recognition rate).

To this direction, few works have been reported in the literature applying a selection mechanism in order to select the most appropriate moments for a specific application. The most of these methods selected moment features in a *wrapper* scheme by taking into account the model (e.g. classifier) that handle the selected moments through the application of an Evolutionary optimization algorithm such as a Genetic Algorithm (GA) [107, 67, 101].

However, the main disadvantage of the GA-based selection is its high computation time for converging to a suitable solution. This drawback makes the *filter* selection methods [101] an attractive alternative approach. These methods do not use the mining model, instead the internal data properties/characteristics (dependency, correlation etc.) are taken into consideration. It is worth noting that this research direction is open, since the need for a fast and adequate moment selection methodology which takes into account the particularities of each application still exists.

1.5.2 Applications

From the previous sections presenting the theoretical aspects of moments and moment invariants can be concluded that the evolution of the moment functions theory has been also motivated by the increased needs of using moments and their invariants in many disciplines. Although the applications of the moments and their invariants increase year by year, there are some fields of applications where moments have provided important solutions when compared with other similar methodologies. The applications where the moments have shown significant impact through the years constitute the subject of this section. This impact is analyzed by presenting the way moments and their invariants are being incorporated.

1.5.2.1 Image Analysis

Although moment functions can be of any dimension, their 2-D realization has found significant applications in image analysis. The ability of image moments to capture the content information of an image in a compact way and with minimum redundancy, makes them appropriate to describe the pixels distribution of the image uniquely. Due to these important properties image moments have been used successfully in texture segmentation [160], image registration [34, 23], sub-pixel edge detection [40, 10], rotation angle estimation [125, 73], image compression [99, 124], image denoising [65, 188], shape analysis [92, 203, 202] and image matching [20]. Recently, image moments have shown promising performance in describing the quality of images [169, 152] by giving a quality index close to human's perception.

1.5.2.2 Pattern Recognition

The main properties of image moments and their invariants are the ability to describe uniquely the pixels distribution and the robustness to geometrical and non-geometrical transformations of the image's content. These two properties make the usage of moments in pattern recognition and classification applications, where some image patterns have to be distinguished, an important utility. The application of moment and moment invariants in pattern recognition origins back to the early works of [60, 28, 1]. Since then, they are applied with remarkable performance on sketched symbol [58], gait recognition [135], target recognition [84], aircraft recognition [28, 191]iris recognition [88, 51], hand gesture recognition [123, 4], facial expression recognition

[75, 66], infrared face recognition [110, 29], human action classification [181], traffic sign recognition [30], texture classification [90].

1.5.2.3 Multimedia Watermarking

The introduction of moments and moment invariants in image watermarking was performed by Alghoniemy [3], who used the Hu's moment invariants and an iterative process in order to hide the watermark into an image. Kim and Lee [72], make a step forward by using the orthogonal Zernike moments to hide the watermark information, in order to take advantage of the rotation invariance and reconstruction capabilities of the radial polynomial basis. A milestone in moment-based watermarking, was the pioneering work of Xin et al. [177], which changed the way the watermark is inserted and extracted. In this approach the dither modulation is applied by adding a "blind" nature to the whole watermarking procedure since the initial watermark was not necessary any more.

Among the several moment-based methodologies [158] incorporating moments as information carriers, new moment families [78, 166, 139, 140, 154, 155], moments with improved local behavior [185, 26, 118, 119], RST (Rotation, Scale, Translation) invariance capabilities [198, 195, 166] and robustness to affine transformations [192] have been introduced lately.

The most emerging topics in moment-based watermarking is the application of the newly proposed quaternion moments as features to hide the watermark information inside color images [157, 156, 167] and the tackling of watermarking as an adaptive process [159, 119, 156, 154, 140] where the significance of each moment coefficient and the image insertion portion are dynamically decided.

Finally, it should be noted that moments have not been used only in image watermarking but in video [161] and audio [165]watermarking as well.

1.5.2.4 Image Retrieval

Nowadays, there are massive amounts of image data due to the many camera equipped devices (computers, mobiles, tablets, etc.) and the unstoppable usage of social media. This high traffic and storage of image data assumes the existence of big image databases, where the task of searching and retrieving images with specific attributes constitutes an every day task for the modern information systems. Therefore, there are needs for accurate, fast and reliable image retrieval systems. Concerning the Content Based Image Retrieval (CBIR) scheme, the usage of efficient image features to describe the content of the images can ensure a high retrieving performance.

Image moments and moment invariants have been used successfully in CBIR image retrieval systems for several years [170, 171, 6, 100, 136, 5]. Moments can describe uniquely the global, as well as the local information [137, 138] of the image's content and thus can be used to distinguish different images and to provide high matching rates of the query image.

Although the application of moment functions as features in retrieving images from a database has been already investigated, there are a lot of new moment families that are not used for CBIR purposes. Moreover, quaternion moments and moment invariants [68, 70] could be promising features to retrieve color images, since they can enclose all the color information in a single hyper complex quantity.

1.5.2.5 Medical Image Analysis

The application of moment functions in medical image analysis follows almost the same directions with the conventional image analysis. The orthogonal moments have been used widely to reconstruct CT images [163, 24] and noisy CT, MRI, X-ray medical images [57], to describe the texture of a CT liver image [9], while several moment invariants have been used as discriminative features to detect tumors [63], to predict protein structures [176], to recognize parasites [27] and spermatogonium [87] and to segment medical images [91, 37].

The acceptable performance of the moments in medical image analysis justifies their ability to describe complex image patterns and generates many expectations for a more systematic incorporation of the recent advances in moments' theory to medical image analysis.

1.5.2.6 Forensics

Image moments, as features describing uniquely the content of an image, have found application to a quite recent topic of information security called *image forgery detection*. Moreover, the invariant properties of moments are showing very useful for the aforementioned task, since they remain unchangeable when common geometric and non-geometric attacks are applied on the image by any malicious user.

More precisely, the orthogonal moment invariants have been used to detect the copy-move image forgery [89, 81, 79, 62] according to which a part of an image is copied and pasted into another place inside the same image, generating duplicate regions. The radial moment invariants can also handle successfully the copy-rotate-move [129, 128] image forgery since they are rotation invariant and thus remain unaffected to the rotation attack.

This research field is still unexplored regarding the application of the moments and their invariants and thus a lot of open issues exist that can be addressed in the future by taking advantage of the recent achievements in the moments' theory.

1.5.2.7 Miscellaneous

Apart from the above applications of moments and moments invariants, there are several other disciplines where the moments have little contribution but the promising results have shown that there is still room for more. Such areas are camera calibration [77], robot localization [76], visual servoing [14], audio content authentication [86, 85], music identification [80], spectral analysis [190, 18], strain analysis [164] and many more.

1.6 New Horizons

The previous analysis of the most emerging research topics both in theory and applications of moment functions having attracted the scientific interest so far, provides a measure of the activity taken place in the last the 50 years in the field of moments and moment invariants. However, the aforementioned analysis also sets the current needs that have to be satisfied through future research by establishing some new horizons. Some distinctive directions that can lead to new lands of innovation and can motivate the early stage as well as the experienced scientists, are presented hereafter.

1.6.1 Local Behavior

Moments and moment invariants, initially were proposed as global image descriptors, since they are computed over the entire image intensity function. This global encoding mechanism makes them robust to noisy conditions since the lower order moments describe the coarse image content, whereas the noise contaminates the image's details. Moreover, the discrimination power of the moment descriptors is distributed over all orders and thus the local information of the image is shared to many components, by making them less efficient to describe the particular properties of a local image region. For example, for textured images, where useful information is highly localized, the global moments are not able to describe the high variability of the texture microstructures.

Some attempts to increase the local description capabilities of the moments have already been reported in the literature for texture classification [90], invariant pattern recognition [114] and image watermarking [26, 119]. However, we are still far away from a theoretical framework that define a local information preserving mechanism during the computation of moment descriptors and more efforts have to be made.

1.6.2 Combined Invariants

As it has already been discussed in Section 1.5.1.4, the moment invariants constitute a useful tool in describing the contents of an image despite the presence of some common geometric and non-geometric deformations. Recently, the embodiment of multiple invariant properties [146, 197, 193, 16] to the moments have become a hot research topic. The development of combined moment invariants to rotation, scaling, translation, blur and affine transformations is still in the beginning and deserves the attention of researchers.

The development of moment invariants robust to multiple geometric and non-geometric image transformations simultaneously and not partially, would show novel directions towards the efficient and unified handling of image deformations.

1.6.3 Selection

One of the most challenging open issue in the theory of moments and moment invariants is the development of an analytical methodology for the selection of the most appropriate moment set for a specific application and image modalities. Currently, a set of moments up to a certain order is computed and used as image descriptors, a practice that does not guarantee the highest performance relative to the problem's objectives. Although, some preliminary approaches [107, 67, 101] have been proposed in the past, as analyzed in Section 1.5.1.8, they have major shortcomings mainly due to high computation time.

The ultimate goal towards the rising of this horizon is the development of a moments and/or moment invariants selection scheme guaranteeing the optimality of the selected features set subject to a specific problem. The development of such a methodology will boost the performance in all the applications described in the previous sections.

1.6.4 Software Library

An exhaustive search in the web can lead to the conclusion that there is not any software package or library in any programming language and environment being dedicated to moments and moment invariants. The majority of the researchers that propose their own algorithms and tools, do not give attention in preparing their source codes in a form suitable to distribute them to the moments community and thus the dissemination of the field is restricted. There is a need to compile a formal and open-source library in an advanced programming environment (MATLAB, C++, Python, R, etc.) by incorporating the major achievements in the theory of moments. In this way, it is strongly believed that the evolution of the field will be boosted, new improved algorithms will be generated and all the algorithms would be compared in the same base.

1.7 Discussion

For the first time, this chapter presents a comprehensive study of the research in the field of moments and moment invariants in the last 50 years. The literature analysis showed the continuously increasing interest in the field of moments but most important that the field's activity is at the zenith so far, a conclusion that can enforce the research in multiple directions.

The previous sections provided an overview of the attainments achieved through the years in each theoretical aspect of moments and where these achievements have found application. As an outcome of the point to point description of the past research actions and considering the current needs of the scientific community, new horizons of research are declared for the future.

This chapter can serve as a guide to the early stage and experienced researchers, who are interested in focusing their research to the moments' theory and their applications.

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