

Moments and Moment Invariants

Theory and Applications

Edited by George A. Papakostas



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Editor

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Foreword

Moments of image intensity distributions have long been used as quantitative measures of shapes present in images. Many years ago, pattern analysis applications found moments valuable for representing image features, and prompted researchers to look for fast and novel methods using different types of kernel functions for efficient shape recognition and classification. The field of moment functions and their invariants then rapidly evolved into an active research area that saw several significant developments in the recent past.

Almost two decades have passed since I wrote my first book on the usefulness of moment functions in image analysis. At that time, the primary applications of moments were limited to invariant pattern recognition, classification, template matching and pose estimation. Geometric, orthogonal and complex moments were used as global shape descriptors in the above domains. Orthogonal moments were the main focus of much research that subsequently led to the development of a new breed of discrete orthogonal moments based on Tchebichef, Hahn and Krawtchouk polynomials. These moments and their corresponding feature descriptors became the subject of rigorous treatment and analysis in the following years.

The fast progress attained towards deriving effective and robust feature descriptors using discrete orthogonal kernels also fuelled the growth in the number of application areas using those moments. We saw a surge in research publications on moment based techniques for image compression, image quality assessment, image watermarking, texture analysis, face recognition and biomedical image processing. Many acceleration methods have also been reported for fast and accurate computation of moments using elegant algorithmic procedures, parallel processing and hardware implementations. The research focus now appears to be slowly shifting towards new variants of the moment functions such as radial and local moments.

In summary, the field of moment functions has witnessed a tremendous growth into a multi-faceted and inter-disciplinary realm of research. This book reflects this growth by providing a comprehensive coverage of important developments in recent years. I greatly appreciate the efforts of Prof. George Papakostas in bringing together a set

of highly relevant and interesting topics in this publication. I have no doubt that the book will prove to be a valuable resource to researchers and stimulate further research in applications involving image moments.

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Preface

A lot of work has been reported in the field of moments and moment invariants, firstly introduced by Hu almost 50 years ago. Through the years an increased interest towards the development of novel moment families, fast and accurate computation algorithms and the application of moments to several disciplines has been presented. In the light of this increased interest, the need for books presenting the main theoretical and practical aspects of moments and moment invariants is in some sense inevitable.

Despite the existence of three notable books written by eminent scientists in the field of moments (R. Mukundan, *"Moment Functions in Image Analysis: Theory and Applications"*, M. Pawlak *"Image Analysis by Moments: Reconstruction and Computational Aspects"* and J. Flusser, B. Zitova, T. Suk, *"Moments and Moment Invariants in Pattern Recognition"*), the aforementioned need still exists. Since the publishing of the above three books, a lot of progress has been reported in considerable journals and conferences. Therefore, the time has come for a new book to cover the up to date progress in the field of moments and moment invariants. The best way to capture the research activity in a field is by selecting high quality work, from different worldwide invited researchers, in the form of chapters of an edited book.

Based on the above motivation, the editor is honored to organize this book where highly active researchers in the field of moments were invited to contribute with recent research works according to their expertise. For the first time researchers in the field of moments get together under the same publication, by contributing the most up to date subjects relative to moments and moment invariants towards the dissemination of established and new ideas to their scientific counterparts.

The book is divided in two parts. The first part, consisting of eight chapters covers theoretical aspects of moment and moment invariants, while the second part presents some traditional and novel applications of moments justifying the multidisciplinary character of the specific research field.

The first chapter titled *"Over 50 Years of Image Moments and Moment Invariants"* written by the editor attempts a survey of the research in the field of moments and moment invariants over the course of the last 50 years. This chapter for the first time

provides a holistic study of the publishing activity, the active research directions and the new horizons in the field of moments and moment invariants.

The second chapter titled "*Accuracy Analysis of Moment Functions*" written by Prof. S. Liao, a pioneer scientist in the field of moments, who along with Prof. M Pawlak have done outstanding research related to the computation accuracy of moment functions. This chapter discusses the accuracy issues occurring when continuous moments defined in the unit circle have to be computed.

The third chapter titled "*Derivation of Moment Invariants*" written by an outstanding research team consisting of Professors H. Shu, L. Luo and J.L. Coatrieux who have contributed significantly to the evolvent of moments' research field. In this chapter an overview of the methodologies for deriving moment invariants under common geometric and non-geometric transformations is presented.

The fourth chapter titled "*Moment Invariants for Image Symmetry Estimation and Detection*" contributed by Prof. M. Pawlak, the author of one of the three pillar books in moments' theory and a pioneer in studying the accuracy of continuous moments defined in the unit circle. This chapter addresses the issue of image symmetry estimation and detection by using moment invariants, a novel research direction and of high importance in image analysis.

The fifth chapter titled "*Image Deconvolution in the Moment Domain*" written by Dr. Barmak Honarvar and Prof. J. Flusser a young and promising scientist and a pioneer researcher in the field of moments with outstanding research especially in affine and blur moment invariants. This chapter proposes a novel framework for image deconvolution in the moment domain.

The sixth chapter titled "*Local Tchebichef Moments for Texture Analysis*" written by Prof. R. Mukundan a pioneer scientist in the field of moments and the author of the first published book in moments. This chapter introduces some new ideas regarding the enchancing of moments abilities to describe local information such as textures.

The seventh chapter titled "2D and 3D Image Analysis by Gaussian-Hermite Moments" written by Dr. B. Yang, Prof. M. Dai, Dr. T. Suk and Prof. J. Flusser a very experienced research team with substantial contributions to the field of moments. This chapter introduces 2D and 3D Gaussian-Hermite moments and their corresponding rotation invariants.

The eighth chapter titled "*Generic Orthogonal Moments and Applications*" written by Dr. C. Camacho-Bello, Prof. C. Toxqui-Quitl and Prof. A. Padilla-Vivanco a highly active research team in the field of moments. In this chapter a detailed analysis of the Jacobi-Fourier moments and their applications in digital image processing is presented.

The ninth chapter titled "*Using Low-Order Auditory Zernike Moments for Robust Music Identification in the Compressed Domain*" written by Prof. W. Li, Dr. B. Zhu, Prof. C. Xiao and Prof. Y. Liu, an experienced research team who have contributed to expanding the applications of moments to other disciplines than imaging. This chapter proposed Zernike moments to solve an important problem of audio analysis the music identification.

The tenth chapter titled "*Image Annotation by Moments*" written by Professors M. Oujoura, B. Minaoui and M. Fakir who constitute a research team recently working in the field of moments. In this chapter the application of image moments in image annotation is discussed.

The eleventh chapter titled “*Should We Consider Adaptivity in Moment-based Image Watermarking ?*” written by Dr. E.D. Tsougenis and the editor discusses some new concepts of applying moments in image watermarking. This chapter highlights the needs for adaptive watermarking in moment domain by discussing possible realizations of adaptivity.

The twelfth chapter titled “*Zernike Moments for Binary and Gray-Level Image Retrieval*” written by Dr. S. Abu Bakar, Prof. M.S. Hitam and Prof. W.N.J. Hj Wan Yussof a quite new research team in the field of moments but with substantial contribution to the application of moments in image retrieval. This chapter presents the fundamentals in applying moment functions in retrieving images from a large database.

The editor would like to acknowledge Prof. R. Mukundan for his support in organizing this book and the authors for their chapter contributions, without whom this book would have been just a wishfull thinking.

Finally, the editor would also like to thank *Science Gate Publishing* for the continuous support and providing of all the necessary resources in order to compile this book keeping its scientific and professional character to a high level.

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