CHAPTER 11

Should We Consider Adaptivity in Moment-based Image Watermarking ?

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The term adaptivity is absent from the state-of-the-art moment-based image watermarking methods. A question to be answered is whether adaptive watermark insertion will guide to the enhancement of image's security (concerning a number of requirements such as robustness, imperceptibility, complexity and capacity). Initially, the term adaptivity is being unfold from different perspectives; the selection of the most qualified coefficients (considering their order and magnitude) for carrying the watermark information; the selection of the most qualified image region for hosting the

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watermark information; and finally the optimum calibration of the quantizer parameters for embedding the watermark information. An experimental justification of the need for adaptivity is being presented, highlighting also the classic tradeoff between imperceptibility and robustness. Furthermore, a number of solutions for each adaptivity perspective are presented along with its corresponding analysis (limitations and future work). To the best of our knowledge, the current chapter constitutes the primary attempt for highlighting/justifying the significance of adaptivity during moment-based watermarking process providing also the readers with a number of tools (adaptivity solutions) that function in gray-scale and color space. Next generation moment-based image watermarking algorithms should consider and benefit from the current adaptivity solutions regarding a high quality security result.

11.1 Introduction

The scope of the present chapter is to highlight the significance of the adaptivity in gray-scale and color image watermarking algorithms functioning in a transform domain. While the specific issue has been treated in multiple ways for classic domains such as Discrete Fourier Transform (DFT) [32], Discrete Wavelet Transform (DWT) [4] and Discrete Cosine Transform (DCT) [23], an investigation for image moments' domain is missing despite the enormous amount of works published during the last two decades [30]. Initially, the term watermarking should be interpreted in order to better comprehend the role of adaptivity in the process. Therefore, according to Hartung and Kutter [8], a watermark is a non-removable digital code, robustly and imperceptibly embedded in the original data, which contains information about the origin, status, and/or destination of the data. In our case the data that hosts the watermark information is a $1 \times 2D$ or $3 \times 2D$ image matrix depending whether we secure gray-scale or color images, respectively. One may raise a question on how the watermark information can be embedded within the image content without visually affecting the host image. In the frequency domain, coefficients estimated from the pixel values are being altered in a way that binary watermark information has been attached to them. In our case, image moments carry the watermark information and from now on, by the term coefficients we will refer to the product of the latter transformation. Multiple quantization methods are applied for attaching the information to the coefficients; Dither Modulation (DM) [2] being one of the most commonly used quantizers in the area and also being the one that the present chapter focuses for addressing the adaptivity issue. All details on DM process will be given and discussed in an upcoming section.

In terms of watermarking, we can relate adaptivity to the following perspectives:

- the selection of the most qualified coefficients (considering their order and magnitude) for carrying the watermark information
- the selection of the most qualified image region for hosting the watermark information
- the optimum calibration of the quantizer parameters for embedding the watermark information

All of the aforementioned with respect to a number of requirements are analyzed in the next paragraph.

In details, as for the first perspective, the order and magnitude coefficient values are strongly connected to the type (coarse or details) and quantity of information they represent, respectively. Consequently, modifying higher order coefficients with large magnitude may result to an extensive distortion of host image's details.

Now considering the second perspective, a highly textured area may enclose greater amount of information without being suspected in contrast with the plain or edged areas where even small interventions can be easily detected. As for the third perspective, adjusting the parameters of the quantizer reflects to the strength the watermark is embedded (i.e. Δ parameter in DM quantizer which is the quantization step). Optimum embedding strength shall prevent the watermarking schemes from situations where the information is overprotected (high quantization step) or even lacks protection (low quantization step).

The satisfaction of the basic requirements (*robustness*, *imperceptibility*, *capacity* and *complexity*) composes the ultimate scope of an image watermarking scheme. The interpretation of the aforementioned requirements helps us compose and provide the readers with a re-defined version of proper image watermarking process through the following Proposition 1:

Proposition 1. *A simple implemented/fast (low complexity) watermarking method incorporates the maximum allowable amount of watermark information (high capacity) to the host image with respect to its perceptual redundancy (high imperceptibility) and its tolerance under geometric or signal processing attacking conditions (high robustness).*

However, the interrelationship of the basic requirements Fig.(11.2) generates the traditional tradeoff existing in image watermarking field where uncontrollable manipulations regarding to one requirement's enhancement possibly leads to an alongside degradation of another one. Undesired watermarking cases where it is believed that adaptivity may eliminate, are collected from the performance evaluation on state-of-the-art moment-based image watermarking methods [30] and presented in Fig.(11.1).

A simple example based on the classic algorithm [40] that takes advantage of Zernike moments (ZMs) for embedding watermark information, is presented in Fig.(11.1). The Peak-Signal-to-Noise-Ratio (PSNR) and the Bit Error Rate (BER) (equations of both of them are later provided) are also included for verifying the algorithm's imperceptibility and robustness performance, respectively. The Fig.(11.1a) and Fig.(11.1b) cases correspond to the results produced from quantizing the higher and lower order moments estimated from benchmark image Lena, respectively. At this point, we need to recall that lower and higher order moments basically represent the coarse and details part of the image, respectively. For case Fig.(11.1a), since higher order moments have been altered, a higher visual quality is achieved but the ability to secure the watermark has been decreased (higher order moments are more vulnerable to attacking conditions). On the contrary, for case Fig.(11.1b), lower order moments satisfied the robustness requirement but affected significantly the image content (the ring effect is visually recognizable and numerically justified by the low PSNR value). Figure 11.1 highlights

(a) PSNR = 56.69 dB and BER = 0.26 (b) PSNR = 36.01 dB and BER = 0.22

Figure 11.1: Benchmark image Lena watermarked based on [40] along with the corresponding watermarks (boosted using histogram equalization for better visualization); quantizing the higher (a) or the lower (b) order moments, we highlight the tradeoff between imperceptibility (PSNR) and robustness (BER).

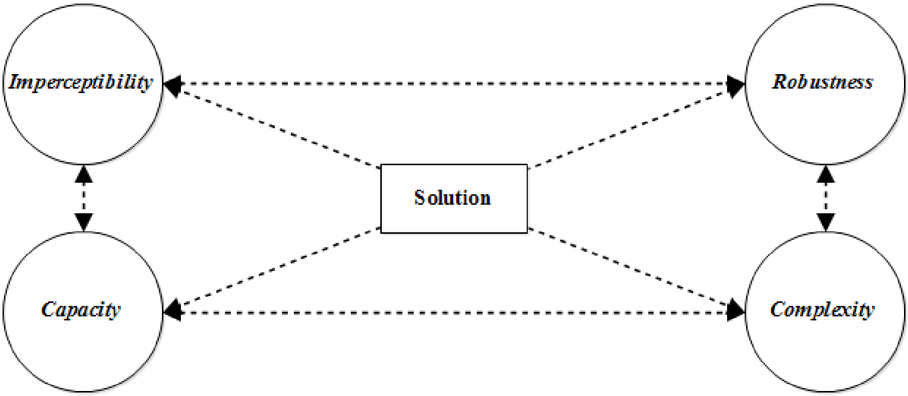

Figure 11.2: The illustration of the interrelationship between basic watermarking requirements.

the classic tradeoff between robustness and imperceptibility requirements existing in the moment-based watermarking algorithms' literature. However, the interrelationship previously discussed is defined between four requirements casting a more complex situation; an attempt to simply illustrate this situation is depicted in Fig.(11.2).

The need for adaptivity in moment-based image watermarking methods has been studied by the authors recently in a series of works [31, 20, 29]. However, each of these works treats adaptivity from a different perspective. The present chapter constitutes the first attempt to gather and study all adaptive moment-based systems for gray-scale and color image watermarking (isolated from the watermarking frameworks). Its scope is to highlight their significance (and consequently adaptivity's too) from all perspectives creating a strong base for the other researchers to benefit and further contribute; it is believed that the present work is just dealing with the tip of the iceberg. In Section 11.2 all related works for moment-based image watermarking are briefly analyzed, while Section 11.3 summarizes the moments types and their properties applied in watermarking. In Section 11.4, the adaptivity issue is partially justified based on a series of experiments that study the tradeoff between robustness and imperceptibility through an optimization process. In Section 11.5, a solution for each aforementioned perspective is presented along with its corresponding limitations or possible future work. Finally, general conclusions are presented in Section 11.6.

11.2 Related Work

Image watermarking constitutes a large area of research interest starting almost two decades ago including works of high contribution in the data security field. All kinds of traditional and recent transformations have been applied regarding high performance solutions. Image moments being one of them have been spread all over the watermarking and authentication community leading to the design of high performance methods. However, the use of image moments has not been restricted to information carriers but it has been extended also to them used as geometric distortion estimators or even descriptors to assess the visual quality of the results. The multi-application of the studied domain in image watermarking area is briefly presented hereafter.

11.2.1 Information Carriers

Based on the-state-of-the-art, the most common use of image moments is for carrying the watermark information. The invariant properties under several geometric and non-geometric image distortions of many of the families made image moments an attractive transformation. Honorably, the first work introducing the moment domain to the watermarking field by Alghoniemy and Tewfik [1] should be mentioned. They applied Hu's [9] seven moment invariants in order to detect the existence of the watermark achieving also robustness to RST (Rotation, Scale and Translation) attacks. Kim and Lee [12] proposed a low complexity and invariant watermarking scheme based on ZMs, but the two detection thresholds that need to be calculated from a large amount of images during the insertion process, constitute a quite laborious task. Pawlak and Xin [21] designed a watermarking method based on LMs but the specific algorithm was time consuming. Xin et al. [40] made a big step in moment-based watermarking field by using the Dither Modulation (DM) [2] which is a special form of quantization index modulation. Although the specific method embeds multiple bit patterns by quantizing the ZMS or Pseudo-Zernike moments (PZMs), still the approximation errors and the computational cost of the corresponding higher order moments do not allow the embedment of large watermark bit messages. Recently, the majority of the researchers focused on the discrete moment families that lack approximation errors and produce higher quality watermarked images. Yap and Paramesran [42] introduced the Krawtchouk moments (KMs) to image watermarking through a low complexity process. Nevertheless, the proposed method lacks rotation invariance and manages to overcome the cropping attack only with the appropriate calibration of watermark's embedding location. Deng et al. [5] introduced a multi-insertion watermarking process which is applied to circular image patches via Tchebichef moments (TMs). Although, the specific method shows resistance to cropping conditions, its complexity is high and possible small errors in the relocation of the patches may lead to significant errors [18] during moments' calculation at the detector's side. Li et al. [13] introduced the Polar Harmonic Transforms (PHTs) in image watermarking field proving their low complexity and high robustness performance. Tsougenis et al. [27] examined the use of Separable moments (SMs) managing to evaluate their performance in image watermarking. Recently, Singh et al. [25] introduced a novel technique for high capacity watermarking scheme using accurate and fast radial harmonic Fourier moments. Tsougenis et al. in [31] managed to take the advantage of the PHTs's properties and eliminate the adaptivity crucial parameters (order, magnitude). To the best of our knowledge, the specific work constitutes the first moment-based image watermarking method that discusses and deals with adaptivity issue (as from the first perspective). Furthermore, the same authors tried to deal with the adaptivity from the other two perspectives. In [20], the introduced technique is making use of a simple Genetic Algorithm for optimizing Krawtchouk moments' parameters achieving this way a high robustness and imperceptibility performance.

At this point, it should be noted that all aforementioned algorithms deal with just gray-scale images. Recently, quaternion image moments designed for color images have been tested in the watermarking application. Wang et al. [34] proposed a robust color image watermarking scheme based on local quaternion exponent moments. The robustness/capacity tradeoff can be highlighted through this work due to the fact that watermarks are selectively embedded in specific areas indicated by feature points. In addition, the nature of each area is not taken into consideration adjusting experimentally the embedding strength (lack of adaptivity). On the contrary, in [29], watermark is embedded adaptively in color images according to third perspective. The color image is split in non-overlap blocks where local quaternion moments are estimated and quantized adapting the embedding strength to block's nature.

11.2.2 Geometric Distortion Estimators

The significant loss of information caused by the geometric/desynchronization attacks has generated the need of a pre-processing step where the geometric distortions are estimated in order to recover the watermarked image to its original form. A number of works [35, 33, 46] have followed the same strategy of the trained Support Vector Machines (SVMs) by image moments in order to estimate the geometric transformations. Although, the specific works present high performance still the complexity of this combination stays in high levels. On the contrary, Li et al. [44] proposed a method that straightforward estimates the rotation angle and the scaling factor of attacked images by only taking into consideration the first three lower order TMs of the original and attacked watermarked images. However, the specific work increases the side information by carrying this number of moments to the detectors side.

11.2.3 Image Quality Assessment

Image moments' high performance in image description inspired Wee et al. [38] to create an image quality assessment metric based on KMs and TMs. As a matter of fact, the application of the specific metric in image watermarking field regarding a better assessment of the produced visual quality results, constitutes the latest innovative use of the studied transformation (trend) [28].

11.3 Image Moments

Image moments are region-based descriptors that correspond to the projection of the image on a specific polynomial base, where the type of the polynomials gives the name to the specific moment family. The computation of a single moment comprises a repetitive process of polynomials evaluation for each image's pixel (Eq.(11.1). Theoretically, the inverse process should lead to a reconstructed image identical to the original one with respect to the maximum order value (Eq.(11.2)). The major categories of the commonly applied moment families in watermarking are presented hereafter:

11.3.1 Geometric Moments

The first introduced moments are the geometric moments where their projection base is defined by "xy" monomials of several orders. However, their base is not orthogonal and therefore their information redundancy is very high.

11.3.2 Continuous Moments

Teague [26] introduced the Zernike (ZMs), Pseudo-Zernike (PZMs) and Legendre (LMs) moments which are orthogonal and manage to overcome the geometric moments' drawback of redundancy. Nevertheless, their computational instabilities [14] especially in higher order values lead to undesirable behaviors. ZMs, PZMs and LMs along with the Wavelet (WMs) [17] and Fourier-Mellin (OFMMs) moments [24] are the most widely referred in the literature moments which are defined in the continuous coordinate space. Meanwhile, Polar Harmonic Transforms (PHTs), one of the most recently introduced transformations [41], have the ability to generate rotation invariant features with no numerical instabilities through higher order values, in a simplified computation framework. PHTs are further divided into three categories, the Polar Complex Exponential Transform (PCET), the Polar Sine Transform (PST) and the Polar Cosine Transform (PCT).

11.3.3 Discrete Moments

Tchebichef (TMs) [16], Krawtchouk (KMs) [43] and dual-Hahn (dHMs) [47] moments being defined in the discrete domain, manage to eliminate the coordinates' normalization that continuous moments need but lack rotation and flipping invariances. However, KMs and dHMs have the significant locality property as being calculated over specific regions adjusted manually by the user.

11.3.4 Separable Moments

Zhu [45] introduced the Separable moments (SMs) which are constructed by combinations of different continuous or discrete orthogonal polynomials. The specific moments' combinations have desirable image representation capabilities and can be useful [28] in image watermarking field.

11.3.5 Quaternion Moments

During the last decades, the quaternion algebra has been widely applied in Discrete Fourier transform (DFT) domain representing a variety of color spaces such as RGB [6], YCbCr [34], CIELAB [32] etc. Despite image moments' close mathematical nature to DFT, the first quaternion moment families have been recently introduced, based on Fourier-Mellin [7] and Zernike [3] polynomials. However, the specific moments families are calculated based on continuous orthogonal kernels which are defined in the polar coordinate system. Although they present minimum information redundancy and rotation invariance, their continuous nature generates approximation errors basically caused by the transformation from Cartesian to polar coordinate system. On the contrary, the usage of discrete orthogonal polynomials eliminates the specific drawbacks and leads to the construction of a promising group of moment families based on Tchebichef, Krawtchouk and dual Hahn polynomials. The discrete moments' lack of rotation invariance has been also satisfied by Mukundan [15] introducing the Radial Tchebichef (RTMs) moments where the corresponding discrete polynomials are transformed in polar coordinates. An extension of the specific family, along with the rest discrete ones to quaternion algebra has been recently presented in [10, 11].

Each moment family is named after the type of the base polynomial used during its calculation process (i.e. dual Hahn Moments (dHMs), quaternion radial Tchebichef Moments (QRTMs)). For the case of separable moments, the produced families adopt both the original used polynomial names (i.e. Hahn Krawtchouk Moments (HKMs)). The polynomials' Kernel function is defined as the orthogonal basis that provides to each moment the ability to describe different part of the image leading to minimum information redundancy.

11.3.6 Moment's Background

Assuming an original image $f(x, y)$ of size $N \times N$; the general equation for calculating orthogonal moments with order n and repetition m has the form:

$$M_{nm} = NF \times \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} Kernel_{nm}(x_i, y_j) \times f(x_i, y_j), \tag{11.1}$$

where $Kernel_{nm}$ corresponds to the moment's kernel consisting of the specific moment family's polynomials of order n and repetition m that constitute the orthogonal basis and NF denotes the normalization factor. The inverse process where the original image can be reconstructed from a finite number of moments up to a maximum order n_{max} and repetition m_{max} is presented in Eq.(11.2).

$$F(x_i, y_j) = NF \times \sum_{n=0}^{n_{max}} \sum_{m=0}^{m_{max}} Kernel^*_{nm}(x_i, y_j) \times M_{nm}, \tag{11.2}$$

where $(*)$ is the conjugation operator.

Readers may refer to the corresponding papers cited above for an extensive analysis on the estimation of each moment family, since only basic theory is being given in the current section for saving space.

11.4 Adaptivity–Experimental Justification

The scope of the present paragraph is to test whether robustness performance of classic moment-based watermarking algorithms may increase maintaining imperceptibility in satisfactory levels. The idea is simple; each watermark bit is assigned to the corresponding coefficient with different embedding strength regarding to BER's decrease and PSNR's stabilization. A Genetic Algorithm (GA) will be applied using a designed fitness function for the specific purpose. GA's scope is to provide the watermarking system with multiple individual embedding strength values for each host coefficient. The optimum result will be compared to the BER performance of the single embedding strength case where the same value of the latter is applied to all host moments; a process commonly followed by state-of-the-art moment-based image watermarking algorithms. It should be noted that the same PSNR value should be achieved for single and multiple embedding strength cases in order to have a fair comparison. However, the DM quantization process should be firstly analyzed since the embedding strength is adjusted by calibrating its quantization step Δ.

11.4.1 Dither Modulation Process

Dither Modulation (DM) is a special form of quantization index modulation [2] which is applied in image watermarking systems in order to assign one bit to each transformation coefficient. Eq.(11.3) shows the application of the DM on the $\hat{A}_{p_i q_i}$moment coefficient:

$$\left|\tilde{A}_{p_i q_i}\right| = \left[\frac{\left|\hat{A}_{p_i q_i}\right| - d_i\left(b_i\right)}{\Delta}\right]\Delta + d_i\left(b_i\right),\ i = 1, ..., L \tag{11.3}$$

$$\tilde{A}_{p_i q_i} = \frac{\left|\tilde{A}_{p_i q_i}\right|}{\left|\hat{A}_{p_i q_i}\right|}\hat{A}_{p_i q_i},\ i = 1, ..., L, \tag{11.4}$$

where d_i is the dither function for the i-th quantizer satisfying $d_i\left(1\right) = \Delta/2 + d_i\left(0\right)$, $d_i\left(0\right)$ belongs to $[0, \Delta]$ range, Δ is the quantization step and $[\cdot]$ is the rounding operation. The security level of the embedded information is controlled by the calibration of Δ parameter. A higher Δ value implies a higher security level and vice versa.

11.4.2 Single Δ vs Multiple Δs

In order to justify the adaptivity issue, a typical moment-based watermarking method has been constructed based on commonly applied moment families (ZMs, PZMs and TMs). The basic steps for message embedment are moments' calculation and DM

Table 11.1: List of signal processing and geometric attacks.

Attack	Level	Description
JPEG	4 levels: 20%, 40%, 60%, 80%	JPEG compression
Median	4 levels: 2x2, 4x4, 6x6, 8x8	Median filtering
Noise	5 levels: 1%, 2%, 3%, 4%, 5%	Addition of random noise
Crop	5 levels: 50%, 40%, 30%, 20%, 10%	Image symmetric crop
Scaling	6 levels: 0.5, 0.7, 0.9, 1.1, 1.3, 1.5	Image resize
Rotation	9 levels: 5, 10, 15, 20, 25, 30, 35, 40, 45 degrees	Rotation by angles from the first quadrant

quantization which are previously discussed. The embedding strength (Δ) is either different (multiple case) or identical (single case) for each moment coefficient. The performance of the robustness requirement is investigated under common signal processing and geometric attacks depicted in Table 11.1. The specific attacking conditions are applied by the well-known benchmark Stirmark [22].

A Genetic Algorithm, a tool commonly used in watermarking systems for solving several optimization problems [19, 20], is applied in order to find the optimum set of Δs (different for each moment participating in the watermarking insertion procedure). The specific optimization process is applied in multiple Δs case searching for a suboptimal set of Δs that will higher satisfy the robustness requirement. The GA's applied settings are: population size 30, maximum generations 40, crossover with probability 0.5 and 2 points, mutation probability 0.01 and *Stochastic Universal Approximation* (SUS) selection method.

The structure of each chromosome of the GA's population is described in the following Eq.(11.5).

$$Ch_i\left(\Delta_1^i, \Delta_2^i, \Delta_3^i, ..., \Delta_n^i\right), \tag{11.5}$$

where Ch_i is the i-th chromosome and Δ_j^i is the j-th Δ value of the corresponding j-th moment coefficient from the total of n coefficients. Each chromosome corresponds to a candidate optimum set of Δ values. The GA finds the necessary number of Δs depending on the message length (bits) and applies them to the method depicted in Fig.(11.3). Thereafter, the traditional measures Peak Signal to Noise Ratio (PSNR) and Bit Error Rate (BER) are adopted for assessing image quality and robustness respectively. Finally, the metrics' results are applied to the proposed fitness function (Eq.(11.6)) in order to evaluate the usefulness of the candidate solutions.

$$fitness = NF_1 \times |PSNR - PSNR_{Target}| + NF_2 \times \left(\frac{1}{T}\sum_{j=1}^{T}(BER)_j\right), \tag{11.6}$$

where T is the number of the attacks applied to watermarked images, NF_1 and NF_2 are the normalization factors for the PSNR and BER respectively and $PSNR_{Target}$ is the specific PSNR value that the process needs to achieve through the genetic

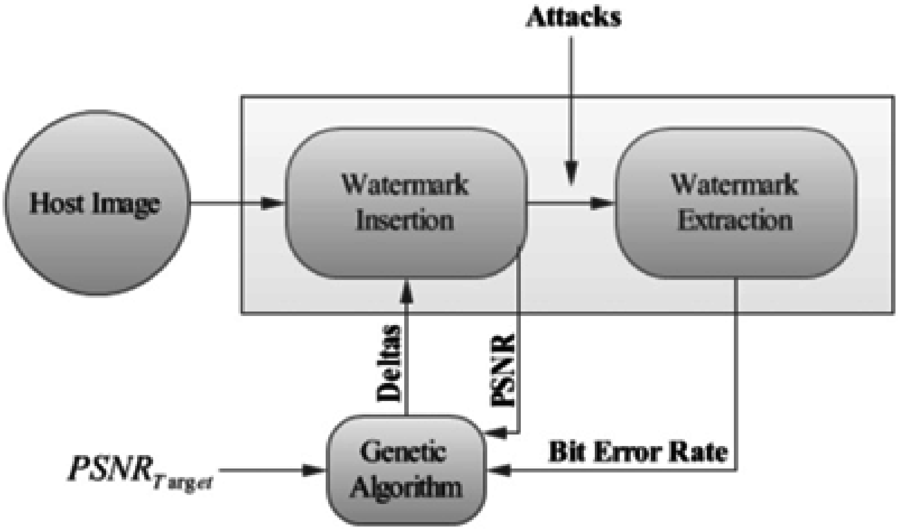

Figure 11.3: The watermarking process in cooperation with the GA.

algorithm. The $NF_1$and NF_2 factors are equal to 10 and 100 respectively due to the fact that the authors need to reach a PSNR value close to the $PSNR_{Target}$ that equals to 45 dB. The PSNR and BER metrics are calculated according to Eq.(11.7) and Eq.(11.8).

$$PSNR = 10\log_{10}\left(\frac{L_{max}\times L_{max}}{MSE}\right),\, where\, L_{max} = 255, \tag{11.7}$$

$$BER = \frac{Number\, of\, Error\, Bits}{Total\, Number\, of\, Bits}, \tag{11.8}$$

where MSE is the well-known *Mean Square Error* of the watermarked image.

The goal of the specific experiment is to create two equal quality (PSNR = 45 dB) watermarked images by embedding the same message (5 bits length) either with single or multiple Δs. In order to create the specific condition, the single case's Δ value is derived through a trial and error process, based on traditional moment-based watermarking methods. Thereafter, mean BERs from both cases are compared regarding the results that will experimentally justify authors' assertion about the necessity of moments' adaptive handling during information embedment. The implemented justification procedure is graphically presented in Fig.(11.3).

The tested benchmark images and the simulation results of the commonly applied moment families (ZMs, PZMs, TMs) for both cases are presented in Fig.(11.4) and Table 11.2 respectively.

Based on simulation results of Table 11.2, it can be clearly concluded that the multiple Δs case outperforms the single one. In multiple Δs case, the mean BER for all moment families are decreased by simultaneously producing the same image quality watermarked images. Therefore, the following Proposition 2 can be deduced:

(a) (b) (c)

Figure 11.4: Benchmark images (a) Lena, (b) Peppers and (c) MRI.

Table 11.2: Mean BER for watermark embedment with single or multiple Δs.

	Lena		Peppers		MRI	
	Single	Multiple	Single	Multiple	Single	Multiple
ZMs	0.14	0.07	0.19	0.09	0.35	0.20
PZMS	0.19	0.14	0.22	0.17	0.30	0.18
TMs	0.21	0.17	0.27	0.25	0.23	0.24

Proposition 2. *The robustness performance of a moment-based watermarking method can be significantly enhanced applying different quantization step to each host coefficient.*

Our initial assertion that assigning different embedding strength to each host coefficient would result to higher performance has now been experimentally proved. From now on, researchers working on moment-based image watermarking should concern adapting to the coefficient's ability to host information. The current chapter also provides multiple systems for adaptive moment-based image watermarking, presented extensively hereafter.

11.5 Adaptivity Solutions

The justification of the need for adaptivity during watermark embedding constitutes the base and motivation for the upcoming solutions. Recalling the three perspectives mentioned in Section 11.1, a number of solutions are presented; one for each perspective. The authors motivated by the worthwhile results in Table 11.2, made the next research step and confronted the adaptivity issue through the upcoming solutions.

11.5.1 First Perspective: Independent of Significant Parameters

A first solution to the adaptivity issue is provided by the authors in [31]. The idea arose after examining the properties of the PHTs recently introduced in the watermarking community showing promising behavior. However, the specific transformation has been used simply as information carrier avoiding any further examination of their properties. At this point, we should highlight the significance of the order and magnitude parameters in moments' estimation. The order and magnitude values represent the quality and quantity of moment's carrying information respectively. Higher level order values and magnitudes correspond to large amount of image details in contrast with lower levels that correspond to small amount of coarse image information. These significance parameters generate the need for special handling of each coefficient which should be adaptively quantized with respect to its order value and magnitude. The scope of the present and also the rest adaptivity solutions is to prevent the watermarking schemes from situations where the information is overprotected (high quantization step) or even lacks protection (low quantization step). A question will be answered shortly is whether a combination of PHTs and DM satisfy the need for adaptivity.

The proposed quantization process, independent of the significant parameters, is developed based on a theorem of Xin el al. [40]. According to the latter, given a user defined image quality ($PSNR_{Target}$) the corresponding Δ value can be straightforward estimated. Consequently, the image quality is strongly connected to the embedding strength as presented in Eq.(11.9). Readers should consider though that the specific equation has been solved, based on properties of circularly orthogonal moments.

$$\Delta_{Gen} = 255\left[\frac{10^{PSNR_{Target}/10}}{24}\sum_{i=1}^{L} NF\right]^{-0.5} \tag{11.9}$$

From the generalized form of Eq.(11.9), where NF corresponds to the Normalization Factor of each moment family (provided in the corresponding papers), one may compute the Δ parameter for every circular moment transformation. The corresponding forms of Eq.(11.9) for the case of the circular moment families are as follows:

$$ZMs \quad : \quad \Delta_{ZM} = 255\left[10^{PSNR_{Target}/10}\frac{\pi}{24}\sum_{i=1}^{L}(n_i+1)^{-1}\right]^{-0.5} \tag{11.10}$$

$$PZMs \quad : \quad \Delta_{ZM} = 255\left[10^{PSNR_{Target}/10}\frac{\pi}{24}\sum_{i=1}^{L}(n_i+1)^{-1}\right]^{-0.5} \tag{11.11}$$

$$PCET \quad : \quad \Delta_{ZM} = 255\left[10^{PSNR_{Target}/10}\frac{\pi}{24}L\right]^{-0.5} \tag{11.12}$$

$$PST \quad : \quad \Delta_{PST} = \begin{cases} 255\left[10^{PSNR_{Target}/10}\frac{\pi}{24}L\right]^{-0.5} & n=0 \\ 255\left[10^{PSNR_{Target}/10}\frac{\pi}{48}L\right]^{-0.5} & n\neq 0 \end{cases} \tag{11.13}$$

$$PCT \quad : \quad \Delta_{PCT} = \begin{cases} 255\left[10^{PSNR_{Target}/10}\frac{\pi}{24}L\right]^{-0.5} & n=0 \\ 255\left[10^{PSNR_{Target}/10}\frac{\pi}{48}L\right]^{-0.5} & n\neq 0 \end{cases} \tag{11.14}$$

where n_i is the order value of the specific moment family and L is the total number of moment coefficients taking part in the watermarking process.

According to Eq.(11.10) and Eq.(11.11) it can be clearly concluded that n (the significance parameter) contributes in the watermarking process through Δ's calculation. The specific connection between order value and the traditional moment families sets strong constraints to the watermarking process. Any scheme trying to embed adaptively information through ZMs or PZMs should seriously take into consideration the order parameter. On the contrary, the calculation of Δ values for PHTs is independent of n (Eqs.(11.12 - 11.14)), an outcome that constitutes a significant observation of this study. Coefficient's magnitude constitutes the second significance parameter and its value represents the amount of information carried by the coefficient. Based on the Δ calculation process of traditional moment families (Eqs.(11.10) - (11.11)), it can be easily concluded that the specific parameter is completely avoided. As a matter of fact, either large ($Magn_1 = 100$) or small ($Magn_1 = 1$) magnitude coefficients which are quantized with the same Δ values (i.e. $\Delta = 3$), lead to overprotection or lack of protection of the embedded bits, respectively. On the contrary, PHTs have the advantage of producing only small magnitude coefficients that basically contain similar amounts of information. The specific behavior has been experimentally justified in [31] where random groups of the same number of PHT coefficients resulted to visually similar images after reconstruction process. Therefore, no additional handling for the second significance parameter is needed for PHTs. In addition, the insecure random selection of host coefficients that previous state-of-the-art algorithms deal with is now eliminated.

11.5.1.1 Limitations / Extensions

Although, the specific solution does not take advantage of the significance parameters for adapting to moment's hosting ability, the elimination of them creates a safe environment that is still considered the first adaptive attempt for moment-based image watermarking algorithms. However, a number of significant limitations should be further discussed. The calculation process of Δ is based on the linearity and symmetry property of the circular orthogonal moments (Appendix A of [31]). As a matter of fact, it cannot be applied to watermarking methods that use discrete orthogonal transformations such as TMs. The transformations' properties that belong to the specific category should be further examined in order to construct a new form of Δ calculation process. The strong connection between the proposed adaptive quantization and traditional metric of PSNR should be highlighted at this point. According to the state-of-the-art, the specific metric is commonly applied assessing the visual quality of moment-based image watermarking schemes. Although different kinds of metrics simulating the HVS properties [20] exist in the literature, the proposed Δ calculation process is strongly connected with the properties of PSNR.

11.5.2 Second Perspective: Optimum Host Area

The solution for the second perspective arises from the use of KMs' locality parameters (p_1, p_2). The latter provide the user with the ability to define the spatial location of

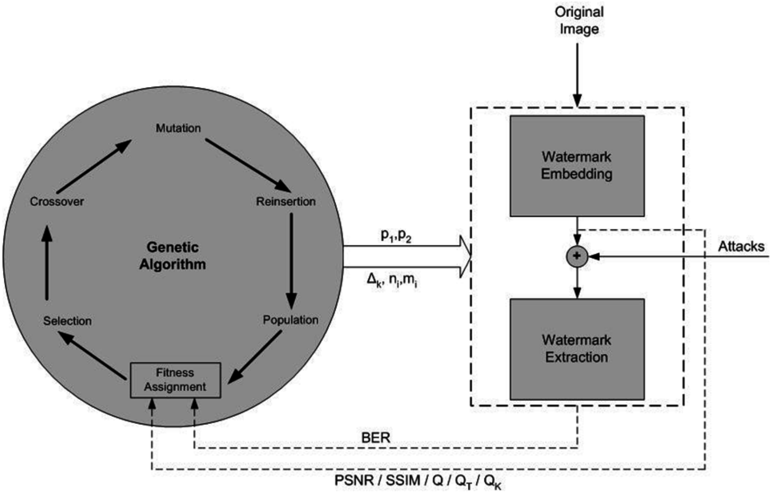

Figure 11.5: Proposed watermarking via genetic optimization.

moment's estimation moving horizontally or vertically within the image range. The specific ability motivated the authors in [20] to transform it into an adaptive process for watermark's insertion. With the help of a simple GA, the most qualified location with respect to robustness and imperceptibility requirements is identified. Consequently, the image watermarking procedure is defined as an optimization problem, which is described by the following formula:

$$\min_{p_1, p_2, \Delta_k, n_i, m_i} (f) \tag{11.15}$$

Based on Eq.(11.15), the ultimate goal is the minimization of an objective function f which is highly dependent on the set of parameters $(p_1, p_2, \Delta_k, n_i, m_i)$. Recalling that p_1 and p_2 are the locality parameters of KMs, Δ_k are the quantization steps of DM (or the embedding strength as concerned the watermarking process) and n_i, m_i are the order and repetition values of moments' estimation. In terms of watermarking, k and i define the number of multiple embedding strengths and the number of estimated moments, respectively. The optimization procedure is illustrated in Fig.(11.5).

As it is depicted in Fig.(11.5), the main processing step dealing with the watermarking procedure is the fitness assignment that measures the appropriateness of a candidate solution (set of parameters). This fitness is assigned by applying specific objective function incorporating the quality of the watermarked image and the fidelity of the extracted watermark information. Before examining the proposed fitness function for optimizing the system, the metrics applied for imperceptibility and robustness assessment should be described. For image assessment, a group of five metrics is composed comprising the classic PSNR [8], the most recent Structural Similarity (SSIM) [37] and Universal Image Quality Index (Q) [36] along also with the moment-based Tchebichef (QT) and Krawtchouk (QK) Moments Quality Index [38]. The visual quality and BER (robustness) results work in concert into the fitness function for evaluating the suitability of the solutions provided by the genetic algorithm.

Although future researchers that follow the specific adaptivity solution may configure the GA in multiple ways, the original settings applied in [20] are: population size 50, maximum generations 50, crossover with probability 0.8 and 2 points, mutation probability 0.01 and Stochastic Universal Approximation (SUS) selection method. The fitness function which is used to evaluate the appropriateness of each candidate solution takes two forms depending on the type of image assessment metric applied.

In this context, the fitness function (F_A) in the case of the PSNR index is defined as:

$$F_A = SF_1 \times |PSNR - PSNR_{Target}| + SF_2 \times \left(\frac{1}{T} \sum_{j=1}^{T} (BER)_j \right), \qquad (11.16)$$

where T is the number of attacks encountered in the procedure, SF_1, SF_2 are scaling factors equal to 10 and 1 respectively, BER_j is the BER of the j-th attacked image and $PSNR_{Target}$ is the desired PSNR value (45 dB). The incorporation of the target PSNR transforms the optimization to a constrained procedure in order to ensure a minimum of image quality that must be acquired.

For the cases including the rest image assessment metrics, the fitness function (F_B) takes the following form:

$$F_B = SF_1 \times |1 - IQ| + SF_2 \times \left(\frac{1}{T} \sum_{j=1}^{T} (BER)_j \right), \qquad (11.17)$$

where IQ is one of the aforementioned quality indices (except for PSNR) and the scaling factors take the same values with Eq.(11.16). Although one can examine all aforementioned metrics, the present chapter encourages the use of SSIM metric, since the experimental results in [20] highlighted SSIM's stable and high performance.

11.5.2.1 Limitations / Extensions

The basic scope of the current adaptivity solution is to identify the optimum image area for hosting the watermark information with respect to robustness and imperceptibility. Despite the significant enhancement of the aforementioned requirements, the current solution suffers from high complexity. The GA takes sufficient time to converge to the optimum solution, a situation that could prevent the extensive use of the specific solution. Moreover, the optimized result is not generic since the optimization process is dependent to each image's content. Specifically, faster and more sophisticated evolutionary optimization algorithms such as Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO) or any other bio-inspired optimization procedure could be applied in order to handle more complex search spaces.

11.5.3 Third Perspective: Optimum Embedding Strength

The current solution manages to produce an individual embedding strength for each 8×8 image block with respect to the latter's nature. Although the specific solution has been proved successful for moment-based color image watermarking, it is believed that similar results will be derived for gray-scale ones. Please consider that color image watermarking is a more complex problem ($3 \times 2D$ matrices to function) and color space properties are not taken into consideration during embedding strength's estimation process. In details, the current adaptivity solution is based on the design of a novel adaptive system that optimizes (offline) a generalized version of the logistic curve based on block's complexity. Then a complementary process (online) adjusts the embedding strength based on the optimized curves regarding to the enhancement of robustness and imperceptibility performance [29]. Therefore, the optimization of the parameters defining this flexible logistic function is considered crucial. The form of the used generalized logistic curve (Richard's curve) is defined as:

$$Y(t) = A + \frac{K - A}{\left(1 + Qe^{-B(t-M)}\right)^{1/v}}, \tag{11.18}$$

where A denotes the lower asymptote, K the upper asymptote, B is the growth rate, $v > 0$ affects near which asymptote maximum growth occurs, Q depends on the value $Y(0)$ and M is the time of maximum growth ($Q = v$).

The optimization process handles the pre-mentioned 6 parameters (A, B, K, Q, M, v) and produces a separate logistic function concerning the block's nature (Plain, Edge and Texture) where the corresponding embedding strength of the block will be defined afterwards. The ultimate "goal" of the proposed algorithm is the adjustment of three kinds of image blocks depending on content's complexity that could be assigned with the appropriate embedding strength based on the optimized logistic curves.

11.5.3.1 Block Classification

Initially, the process that defines the complexity of every 8×8 pixels sized carrier block should be presented. Based on a block classification method proposed in [39], the complexity of its host block is further analyzed in the moment domain. The image should be first converted to gray-scale space in order to block-wisely apply the traditional canny edge detector. The scope of this method is to estimate the edginess (p_{edgel}) of each block based on Eq.(11.19) which will be used to estimate its corresponding embedding strength in the upcoming step (Eq.(11.20)). Based on two pre-defined thresholds (α, β), each block is classified according to the following analysis:

$$p_{edgel} = \frac{N_{edgels}}{(N_{block})^2}. \tag{11.19}$$

where N_{block} is the size of the block and N_{edgels} the number of block's edge pixels.

Considering the aforementioned edginess measure a block is characterized as follows:

$$BlockType = \begin{cases} Plane, & 0 \le p_{edgel} \le \alpha \\ Edge, & \alpha < p_{edgel} \le \beta \\ Texture, & \beta < p_{edgel} \end{cases}. \tag{11.20}$$

The threshold values (α, β) are empirically assigned as 0.1 and 0.2, respectively [39].

The presented analysis quantifies blocks' content complexity based on the number of contained edge pixels (edgels), a measure that is aimed to be correlated with the blocks' embedding strength during the optimization process.

11.5.3.2 Optimization process

A data set of 150 image blocks of 8×8 pixels size (50 for each block category) is provided to the GA regarding to the optimization of the generalized logistic curves. The GA produces 18 parameter values (6 per block category / logistic curve) aiming to minimize the BER and alongside maximize the PSNR enhancing the robustness and imperceptibility system's performance, respectively. The structure of the i-th algorithm's chromosome is defined as:

$$Ch_i \left(f_1^1, f_2^1, f_3^1, ..., f_6^1, f_1^2, f_2^2, f_3^2, ..., f_6^2, f_1^3, f_2^3, f_3^3, ..., f_6^3\right), \tag{11.21}$$

where $\left\{f_1^k, f_2^k, f_3^k, ..., f_6^k\right\}$ with $k = 1, 2, 3$ (1: plane, 2: edge, 3: texture) are the six free parameters of the k-th logistic curve. Each chromosome corresponds to a candidate optimum set of parameter values constructing the three logistic curves (one for each block category).

It is worth noting that the configuration of the applied GA is as follows: population size 20, maximum generations 50, crossover with probability 0.6 and 2 points, mutation probability 0.01 and Stochastic Universal Approximation (SUS) selection method. Moreover, the optimized fitness function (F_A) is identical to that incorporated in the experiments of the second perspective.

The derived optimized logistic curves are considered for adjusting the appropriate embedding strength of each image block according to the following form.

$$\Delta = \begin{cases} Y_{Plane}\left(p_{edgel}\right) & 0 \leq p_{edgel} \leq \alpha \\ Y_{Edge}\left(p_{edgel}\right) & \alpha < p_{edgel} \leq \beta \\ Y_{Texture}\left(p_{edgel}\right) & \beta < p_{edgel} \end{cases} \tag{11.22}$$

The Δ factor which is the quantization step of DM constitutes the embedding strength of the proposed moment-based watermarking scheme. As a matter of fact, the curve defined Δ values are provided to the watermarking framework in order to examine its performance. These steps constitute an offline iterative process (Fig.(11.6)) that terminates when all GA's generations are accomplished.

The best results considering BER and PSNR values indicate the optimum forms of the logistic curves which are provided to the online part of the framework gaining significant time. The proposed adaptive process has been evaluated by comparing the adaptive Δ case ($A\Delta C$) to the single Δ case ($S\Delta C$) where the same Δ value is applied to each host block ignoring the blocks' complexity factor.

The quaternion radial moment families (RTMs, RKMs and RdHMs) and a group of signal processing/geometric attacks are applied in order to test the robustness of the proposed system. Results in [29] (especially for RdHMs) clearly indicated that $A\Delta C$ outperforms $S\Delta C$ in terms of robustness. Moreover, the fact that the results are produced based on random blocks of different complexity makes the system performance stable and independent of the image content (generic solution). Since moment-based

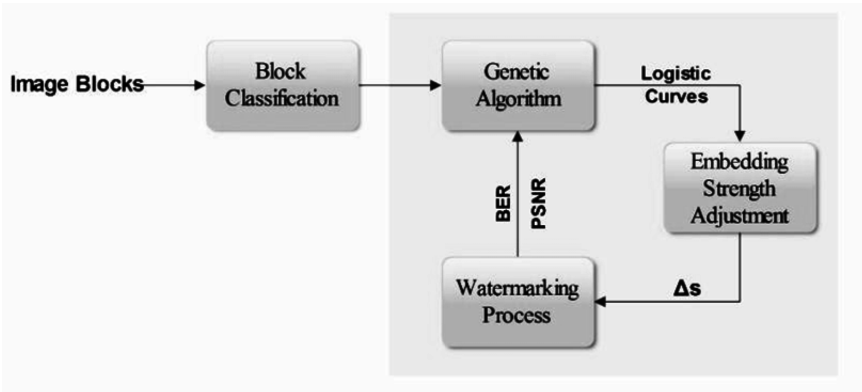

Figure 11.6: Offline operational mode of the proposed adaptive system.

color image watermarking is at its first steps and adaptivity has been proven as a contributing step, future researchers should be motivated and adopt the current system enhancing this way their algorithm's performance.

11.5.3.3 Limitations / Extensions

Although the current solution manages to alongside contribute in the enhancement of the robustness and imperceptibility requirements, the heavy load of computations strictly connected with QRMs estimation leads to a sacrifice of the complexity requirement. However, the block based approach followed by our algorithm avoids estimating higher order moments which are time consuming but still the need for faster and more efficient methods for QRMs' computation should be expected in the near future. Having empirically selected Richard's curve for our solution, there exist multiple other curves that future researchers may examine searching for a more convenient one that could better fit to blocks' nature. It should be noted also that the adaptive scheme has been tested only for the image moment's domain. However, there are no restrictions/limitations applying it in different domains such as DCT, DWT and QFT; it is believed that future transform domain watermarking algorithms may adapt this system and benefit from its promising behaviour.

11.6 Conclusion

One of the most significant research "gaps" in the progress of moment-based image watermarking methods is the lack of coefficient's adaptive handling during the information embedding process. Initially, the current chapter interprets and justifies experimentally the adaptivity issue. Authors propose treating adaptivity from three different perspectives; each one followed by the corresponding solution. The presented adaptive systems (solutions) focus on eliminating the significance moment estimation parameters, identifying the most qualified area and generate blockwisely an optimum embedding strength with respect to area's nature based on PHTs, KMs and RdHMs properties, respectively. All results presented extensively in the original works [31, 20, 29] indicated that significant enhancement of robustness can be achieved followed by high level imperceptibility performance. Next generation of state-of-the-art methods should adopt and benefit from the presented adaptivity schemes. Conclusively, it is believed that the connection between image watermarking, moments and adaptivity has been established. The 'goal' of the current chapter is to stimulate researchers working on moment-based image watermarking area to consider adaptivity.

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